PROBLEM SOLVING MAY SHED SOME LIGHT

Inspired by John Mason, a professor at the Open University who is an admirer of JGB, and teaches 'thinking mathematically'.

Most of my discourse on the dimensions of time can suspend us in metaphysical limbo. It's easy and understandable to feel at a loss. One thing we have to be on the lookout for is going into an abstraction to resolve questions we have arising from the contradictions of life and then leaving life behind and getting lost in the abstraction. One thing we have to do is to 're-enter' where we started from.

What I'm casually calling 'abstraction' can be looked at as a face of eternity. Because the word abstraction can imply being non-actual. So we might consider some process in which we leave the actual, pass through the eternal and return to the actual again. This need not be some mystical quest. In fact I believe it is inherent in *problem-solving.*

Case of the Palindrome

A palindrome number is like 2332 or 7557, the same backwards as forwards. A conjecture arises that every such number is divisible by 11. Is this true?

Well, one can go through every one of the 90 that are possible and check it out. This can be called 'learning the hard way' and it fails to give us any understanding.

In problem solving we must have (a) what we know, and (b) what we want. The latter comes from values: we want to understand, to feel convinced, to discover something new, etc. WHY is every palindrome divisible by 11? We look into their *pattern.*

Think of each particular case as an *actualisation.* We now want to 'see into eternity' a bit. The eternal pattern of the palindrome is written as ABBA, but what does this mean?

This can be done in various ways. We are not sure which is going to pay off. We feel there might be more elegant ways than others, or that there is a pattern that we can extend beyond this particular class of numbers. What we do is try out various ways of relating the general with the particular.

For example, we see there are two variables, A and B. If we change B by adding 1we see that this adds 110 to the number which is of course divisible by 11. If we add 1 to A, this adds 1001 which is also divisible by 11. So, if any ABBA is divisible then all the rest are.

I hope you can see that this sort of move is a combination of the actual and the possible. What we are NOT doing is claiming that we just 'see' or intuit the truth of the conjecture. No, we are after *proving* it must be so. A proof articulates an eternal pattern, I'd like to say. There are many proofs and the American mathematician

Chaitan says that we only *understand* a mathematical idea if we can create *our own* proof for it.

Another move is to spell out what the palindrome form ABBA means. Well, this is an assembly of units, tends, hundreds and thousands (the number 2332 reads 'two thousand, three hundred and thirty two' for example). So:

ABBA = A,000 + B00 + B0 + A = A(1001) + B(110) = 11 (91A +10B)

So *obviously* such palindromes are always divisible by 11. We are getting nearer to some perfectly clear insight.

I would add that 'perfecting' seems to me to be appropriate for the dimension of hyparxis. In the given case, we come to some understanding of *why* palindromes have the property but this is not strong enough to deal with extensions such as the properties of numbers like ABCCBA. Mathematics is essentially hyparchic, seeking patterns in the complexity of actual cases – of numbers, maps, knots, surfaces, sets, whatever.

Problem Solving and Hazard

When one tackles a problem one does not know in advance how to solve it. One of the most remarkable strategies or 'methods' of problem solving I came across seems ridiculously simple and obvious but is very deep (devised by a vicar in the 19th century to solve mazes).

1. Try to solve the problem somehow
2. Register and accept when you get stuck
3. Reverse your steps
4. Until you reach a point at which you could have done something differently
5. Do something different and proceed as before.

I think it is fairly clear that what we do in solving problems is to make something visible that was not seen before. Hence the AHA! moment. As John Mason points out so well in his book *Thinking Mathematically* there are three intertwining aspects (what's in brackets :

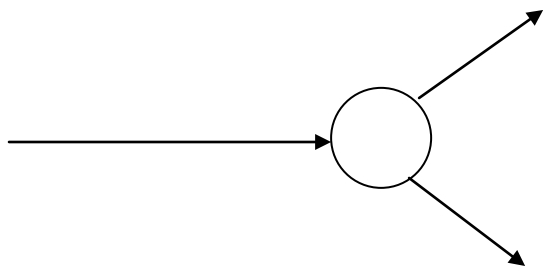
Manipulating (doing stuff – actualisation)

Getting a sense of pattern (becoming more conscious – eternity)

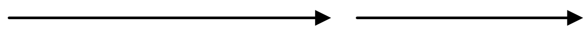
Articulating the pattern symbolically (making visible the invisible – hyparxis)

Hazard is relevant in many senses. There is the obvious sense of just making a mistake (wrong calculation) but also the not so obvious senses of looking to the wrong pattern or misconstructing the articulation. The choice of what to articulate and how is crucial.

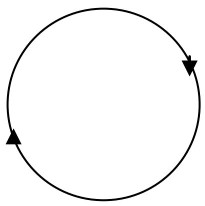
The problem-solving strategy outlined above exemplifies the operation of *questioning assumptions*. This is self-reflective and implies true consciousness. I want to expand this point further and say that it exemplifies *making a gap* that makes it possible to change direction. A ‘gap’ has the boundaries of choice as suggested in the diagram below.



There is usually no choice or the possibility is not seen or felt, shown diagrammatically below as just carrying along the tram lines of time past and time future.



which is really going in a circle and just repeating the same error.



Perspicacious readers may care to reflect on the first diagram run backwards as a thought experiment: what would it mean?

To articulate is hyparchic.

Get rid of the word if it gets in the way. Enjoy the poets:

“What might have been and what has been points to one end which is always present”

But maths problems give us a chance to *experiment.*